

Five Concepts of Net National Product: Supplementary Material

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Introduction

This short supplementary document contains mathematical workings for several points introduced with the words "it can be shown that" within the presentation of the fifth NNP concept. All are straightforward applications of elementary calculus.

Workings 1

We are given the initial stock of capital $K_0 = 1000$ and the equation of motion:

$$\frac{dK}{dt} = -0.1K$$

Differentiating $\ln K$ with respect to time using the chain rule:

$$\frac{d(\ln K)}{dt} = \frac{d(\ln K)}{dK} \frac{dK}{dt} = \frac{1}{K} \frac{dK}{dt} = -0.1$$

Integrating with respect to time:

$$\ln K_t = -0.1t + c$$

Substituting $t = 0$, the constant of integration $c = \ln 1000$ and so:

$$\ln K_t = -0.1t + \ln 1000$$

Taking the antilog of both sides:

$$K_t = 1000e^{-0.1t}$$

Workings 2

Given discounted utility $U_t = 100e^{-0.1t}$ we want to obtain W_0 by integrating this from the present to infinity (note that $e^0 = 1$ and $e^{-\infty} = 0$):

$$W_0 = \int_0^{\infty} 100e^{-0.1t} dt = \left| -1000e^{-0.1t} \right|_0^{\infty} = -1000e^{-\infty} - (-1000e^0) = 0 - (-1000) = 1000$$

Workings 3

We want to find x such that:

$$\int_0^{\infty} x e^{-0.05t} dt = 1000$$

Evaluating the integral we have:

$$\left|_{-20xe^{-0.05t}}^{\infty}\right| = 1000$$

$$-20xe^{-\infty} - (-20xe^0) = 0 - (-20x) = 20x = 1000$$

$$x = 50$$
